Module 12 Probability & Statistics

Module title	Probability & Statistics
Module NFQ level (only if an NFQ level can be demonstrated)	6
Module number/reference	BSCH-PAS
Parent programme(s)	Bachelor of Science (Honours) in Computing Science
Stage of parent programme	Stage 2
Semester (semester1/semester2 if applicable)	Semester 1
Module credit units (FET/HET/ECTS)	ECTS
Module credit number of units	5
List the teaching and learning modes	Direct, Blended
Entry requirements (statement of knowledge, skill and competence)	Learners must have achieved programme entry requirements.
Pre-requisite module titles	BSCH-FC
Co-requisite module titles	None
Is this a capstone module? (Yes or No)	No
Specification of the qualifications (academic, pedagogical and professional/occupational) and experience required of staff (staff includes workplace personnel who are responsible for learners such as apprentices, trainees and learners in clinical placements)	Qualified to as least a Bachelor of Science (Honours) level in Computer Science or equivalent and with a Certificate in Training and Education (30 ECTS at level 9 on the NFQ) or equivalent.
Maximum number of learners per centre (or instance of the module)	60
Duration of the module	One Academic Semester, 12 weeks teaching
Average (over the duration of the module) of the contact hours per week	3
Module-specific physical resources and support required per centre (or instance of the module)	One class room with capacity for 60 learners

Analysis of required learning effort		
	Minimum ratio teacher / learner	**Hours**
Effort while in contact with staff		
Classroom and demonstrations	1:60	36
Monitoring and small-group teaching		
Other (specify)		
Independent Learning		
Directed e-learning		
Independent Learning		50
Other hours (worksheets and assignments)		39
Work-based learning – learning effort		
Total Effort		125

Allocation of marks (within the module)					
	Continuous assessment	**Supervised project**	**Proctored practical examination**	**Proctored written examination**	**Total**
Percentage contribution	30%			70%	100%

Module aims and objectives

This module aims to support learners as they develop a broadly based, and intellectually challenging framework in the area of Probability & Statistics. Learners have an awareness of current statistical techniques, literature, and research in the area. Learners are expected to apply the principles of probability and statistics to solve problems and inform decision making. Learners achieve this through developing knowledge and understanding of probability and statistical principles, while applying these principles in typical real-world scenarios.

Minimum intended module learning outcomes

On successful completion of this module, the learner will be able to:

1. Calculate the probability of a given event
2. Differentiate between random and non-random events
3. Explain how unbiased sampling may be conducted
4. Prepare and present summary statistics
5. Explain and apply the Central Limit Theorem
6. Work with data distributions to solve statistical problems
7. Estimate confidence intervals
8. Conduct significance tests
9. Solve problems of permutations, combinations and derangements

Rationale for inclusion of the module in the programme and its contribution to the overall MIPLOs

As data is the material that computers process, this module supports learners in understanding the nature of data, the techniques used to analyse it and how to derive knowledge from it. This module aims to support learners as they develop a broadly-based understanding of probability and statistical techniques and methods that can be usefully applied in the field of Computer Science. The module aims to support learners to apply the principles of probability and statistics to help solve problems and support informed decision making. Appendix 1 of the programme document maps MIPLOs to the modules through which they are delivered.

Information provided to learners about the module

Learners receive a programme handbook to include module descriptor, module learning outcomes (MIMLO), class plan, assignment briefs, assessment strategy and reading materials.

Module content, organisation and structure

Events, Outcomes & Probability

- Probability calculation
- Rules of probability
- Random and non-random events
- Dependent events.
- Conditional probability
- Independent events.
- The probability tree.
- Baye's Theorem

Summary Statistics

- Finding the mean, mode, median, variance and standard deviation for a set of data
- Presentation and notation

Sampling

- Finite and infinite populations
- Selecting samples
- Calculation of sample mean and variance
- The central limit theorem

The Normal Distribution

- Characteristics of the Normal Distribution
- Measures of location and dispersion

- The characteristics of the Standard Normal Distribution
- The Central Limit Theorem

Confidence Intervals

- Finding a confidence interval for a parameter
- Finding a confidence interval for a proportion

Hypothesis Testing

- Formulation of the Null and Alternative hypotheses
- Levels of significance
- One and two-tailed tests
- Type I and Type II errors
- The use of T-tests

Other Probability Distributions

- The T distribution
- The Poisson distribution
- The Binomial distribution
- Typical distribution application scenarios
- Probability density and distribution functions

Permutations & Combinations

- Calculating combinations, permutations and derangements

Module teaching and learning (including formative assessment) strategy

The module is delivered through a combination of lectures and associated worksheets. The learners complete the worksheets throughout the module. The worksheets are directly related to the material covered in lectures. The emphasis is on developing a broad understanding of data, probability and statistical methods that contributes to learner development in the area of Computer Science. It achieves this through a presentation of a wide range of relevant material, interactive discussion and practical implementations of the methods and calculations presented. A problem-solving approach is emphasized throughout with examples that are solved in class mirrored in the tasks presented in worksheets.

Assessment is divided into two. First there are a number of worksheets that will build the learners skills with the modules content. Finally, there is an end of semester exam that tests the learners understanding of the theoretical material.

Timetabling, learner effort and credit
The module is timetabled as one 3-hour lecture per week.

Continuous assessment spreads the learner effort to focus on small steps, and build up their competence in applying statistical and probabilistic techniques to problems.

There are 36 contact hours made up of 12 lectures delivered over 12 weeks with classes taking place in a classroom. The learner will need 50 hours of independent effort to further develop the skills and knowledge gained through the contact hours. An additional 39 hours are set aside for learners to work on worksheets and assignments that must be completed for the module.

The team believes that 125 hours of learner effort are required by learners to achieve the MIMLOs and justify the award of 5 ECTS credits at this stage of the programme.

Work-based learning and practice-placement
There is no work based learning or practice placement involved in the module.

E-learning
The college VLE is used to disseminate notes, advice, and online resources to support the learners. The learners are also given access to Lynda.com as a resource for reference.

Module physical resource requirements
Requirements are for a classroom for 60 learners equipped with a projector.

Reading lists and other information resources
Recommended Text
Brase, C. H. and Brase, C. P. (2018) *Understanding Basic Statistics*. Boston: Brooks Cole.

Secondary Reading:
Boslaugh, S. (2013) *Statistics in a Nutshell*. Farnham: O'Reilly.

Francis, A. and Mousley, B. (2014) *Business Mathematics and Statistics*. Andover: Cengage Learning.

Lakin, D. S. (2011) *How to Use Statistics*. Harlow: Prentice Hall.

Mendenhal, W. (2006) *Introduction to Probability & Statistics*. Pacific Grove: Thomson Learning.

Specifications for module staffing requirements

For each instance of the module, one lecturer qualified to at least Bachelor of Science (Honours) in Computer Science or equivalent, and with a Certificate in Training and Education (30 ECTS at level 9 on the NFQ) or equivalent.. Industry experience would be a benefit but is not a requirement.

Learners also benefit from the support of the programme director, programme administrator, learner representative and the Student Union and Counselling Service.

Module Assessment Strategy

The assignments constitute the overall grade achieved, and are based on each individual learner's work. The continuous assessments provide for ongoing feedback to the learner and relates to the module curriculum.

No.	Description	MIMLOs	Weighting
1	5 equally weighted worksheets Worksheet1: Learning outcomes 1, 2, 3 Worksheet2: Learning outcomes 1 - 5 Worksheet3: Learning outcomes 1 - 7 Worksheet4: Learning outcomes 1 - 9 Worksheet5: Learning outcomes 1 - 10	1-9	30%
2	Written exam that tests the theoretical aspects of the module	1-9	70%

All repeat work is capped at 40%.

Sample assessment materials

Note: All assignment briefs are subject to change in order to maintain current content.

Worksheet 1

1. (a) What is the probability that the result will be an even number less than 6 on a throw of a six-sided die?

(b) If the probability of correctly guessing the answer to a multiple choice question is 0.25, what is the probability of an incorrect guess?

2. What is the probability that on two throws of a die the result each time will be a prime number?

3. What is the probability of throwing a 3 followed by an even number greater than 3 with a six-sided die?

4. What is the probability of drawing a diamond or a red picture card from a well-shuffled pack of 52 cards?

5. What is the probability of drawing a black picture card from a well-shuffled pack?

6. Show clearly how to calculate the probability of drawing two kings without replacement from a well-shuffled pack?

7. On a roll of two dice, what is the probability that the sum of the numbers on the upper faces is:

(a) 3

(b) 7

8. A coin is thrown three times in succession. What is the probability of getting exactly one head?

9. A coin is thrown three times in succession. What is the probability of getting tails at least twice?

10. On a throw of four fair coins, what is the probability of getting two heads and two tails?

WORKSHEET 2: CONDITIONAL PROBABILITY/SUMMARY STATISTICS/THE NORMAL DISTRIBUTION

Question 1

A previous survey shows that a machine making plastic components is correctly set up for the day's production on 95% of days. On days when it is set up correctly, 98% of the components produced are good. If the machine is not set up correctly, only 40% of the components produced are good. On a particular day, the machine is set up and the first component produced is found to be good. What is the probability that the machine is set up correctly?

Question 2

A wheel bearing factory rejects bearings if they are either oversize or undersize. The probability that a bearing is oversize is 0.004 while the probability that it is undersize is 0.01.

(a) Are oversize rejection and undersize rejection events mutually exclusive?
(b) What is the probability that a bearing selected at random is not within size limits?
(c) What is the probability that a bearing selected at random is correctly sized?

Question 3

In Question 2, bearings are rejected if they are outside size limits. Incorrectly sized bearings are placed in a bin. If a bearing is selected at random from the rejects bin, what is the probability that it is undersize?

Question 4

A company manufacturing disk drives has two production plants located in Singapore and China. The Singapore plant contributes 40% of output with the balance coming from China. Over a prolonged period, it is observed that 5% of the Singapore drives are faulty while 7% of the Chinese produced drives are faulty. All drives are returned to the company's headquarters in Dublin for testing and dispatch. What is the probability that a drive found faulty at this check comes from China?

Question 5

A TV game show involves picking a coloured ball from one of two boxes. A blindfolded contestant chooses a box and then picks a ball at random from the box. Box 1 contains 2 green balls and 8 red balls. Box 2 contains 5 green and 2 red balls. If a contestant picks a red ball, what is the probability that the contestant chose from Box 1?

Question 6

Show, with detailed working, how to find the mean, mode, median, interquartile range and standard deviation for the following set of data:

$$\{28, 31, 6, 9, 32, 21, 42, 37, 45, 47\}$$

Question 7

A study of data collected at a light bulb factory shows that a batch of 6000 light bulbs have a mean life of 1000 hours with a standard deviation of 80 hours. Assuming a Normal Distribution, estimate how many bulbs will fail before 900 hours.

Question 8
If x is normally distributed with u = 25 and s = 7 determine the probability that a randomly selected x lies between 18 and 32.

Worksheet 3: Confidence Intervals 2

Question 1
A survey of 450 randomly selected Irish households shows that only 72% have a broadband connection. Infer a 95% confidence interval for the proportion of all Irish households that have broadband connection.

Question 2
At a vehicle testing centre, 45 out of 200 randomly selected vehicles are found to have defective tyres. Infer a 95% confidence interval for the proportion of all vehicles presenting for testing that have defective tyres.

Question 3
A survey of mortgage holders at the current time shows that 17% are in arrears with their mortgage repayments. Out of the 300 people surveyed, 48% report that they are currently able to manage, while some in the sample also report that although they are not in arrears, they fear for the future. Infer a 99% confidence interval for the proportion of all mortgage holders who are currently in arrears with their mortgage repayments.

Question 4
In a low-temperature performance test (CCA test) of 170 vehicle batteries, an average current reading of 160A is observed, with a standard deviation of 14A. Infer a 95% confidence interval for the current output of a battery selected at random.

Question 5
A television programme is watched by 35% of a representative test audience consisting of 400 people. Infer a 95% confidence interval for the proportion of all viewers who watched the programme.

WORKSHEET 4: COMBINATORICS, SAMPLING, DISCRETE DATA & HYPOTHESIS TESTING

Question 1

(a) Show how to calculate the number of ways that a team of 3 people can be selected from a group of 10?

(b) If in a certain country, car registrations consist of 3 upper case letters followed by four numbers, how many registrations are possible?

Question 2

(a) Explain why it is difficult to pinpoint a precise figure for the mean in a large population. How can the mean be determined?

(b) Explain:

(i) Systematic sampling

(ii) Convenience sampling

Question 3

A machine requires all five of its microcontrollers to operate correctly in order to pass acceptability tests. The probability of the installed microcontroller type operating correctly is 0.99.

(a) What is the probability that the machine passes acceptability tests?

(b) The machine is redesigned so that the five existing microcontrollers are replaced by just three newer chips. The newer type of microcontroller has a probability of operating correctly of 0.98. Determine whether the new design is more reliable than the original.

Question 4

(a) Why is hypothesis testing necessary? What precisely is the objective of a hypothesis test?

(b) A machine fills packets of peanuts where it is stated on the packet that the mean weight is 40 grams. A random sample of 50 packets is taken and the mean weight is found to be 38.9 grams with a standard deviation of 2 grams. Carry out a significance test at the 5% level.

Question 5

(a) In relation to hypothesis testing, what is a one-tailed test? Give an example of where a one-tailed test might be used.

(b) A car manufacturer claims that its cars are capable of travelling 9 kilometres per litre of fuel. Test runs with 90 cars result in an average distance travelled per litre of fuel of only 8.2 kilometres, with a standard deviation of 2.5 kilometres. Carry out a one tailed test at the 5% level to test the manufacturers claim.

Worksheet 5

Question 1

(a) Outline a method of selecting three random numbers in the range 1 to 6.

(b) Library methods such as Java's Math.random(), return a pseudo-random number rather than a truly random number. Explain clearly why this is the case.

Question 2

A computer component manufacturer creates serial numbers for its products using the digits 0, 1, 2, 3, 4, 5, 6, 7 and 8. How many valid seven digit numbers can be created if there is no repetition of digits and a seven digit number may not start with zero?

Question 3

A microprocessor manufacturing facility produces 300 microprocessors per hour. The probability that an individual chip is faulty is 0.01. Calculate the probability that in a given hour's production:

(a) Two chips are faulty
(b) Not more than 3 chips are faulty.

Note: This is Binomial data but since n is large and p is small, the Poisson distribution is a good approximation to the Binomial distribution. In this case, either approach yields approximately the same answers.

Question 4

(a) Is a computer password a combination or a permutation? Explain.

(b) A company introduces a computer password system for a local area network which enforces the use of passwords consisting of 8 alphanumerics. More precisely, passwords must consist of 5 lower case letters and 3 digits in any order without repetition. How many different passwords are possible using this system?

Question 5

(a) Internet Protocol Version 4 (IPV4) addresses consist of 32 bits. The addresses can be thought of in their more human readable form as composed of 4 8-bit numbers. How many unique addresses (including zeros) are possible with IPV4?

(b) Internet Protocol Version 6 proposes network addresses consisting of 128 bits. How many unique IP addresses are possible with IPV6?

Question 6

An examination contains ten multiple choice questions. Each question has five options only one of which is selected by the examination candidate. Calculate the probability that an examination candidate who simply guesses the answers will get at least three questions correct.

Question 7
Explain, with the help of one example in each case, what types of problems knowledge of the following data distributions is helpful in solving. In particular, explain the precise nature of the data in each case.
(a) Poisson Distribution (b) Binomial Distribution

Question 8
A minibus has 9 passenger seats. The probability of a seat being occupied is estimated to be 0.63. Calculate the probability that on a typical run:

(a) there are no passengers;
(b) there is just 1 passenger;
(c) there are exactly 2 passengers;
(d) there are at least 3 passengers.

Question 9
Calls at a call centre are observed to arrive at a mean rate of two per minute. What is the probability of receiving three calls in a minute?

Question 10
Observations at a certain large organisation show that on average, there is a server outage every 6 months. What is the probability of getting through a year without any server outage at all?

GRIFFITH COLLEGE DUBLIN

QUALITY AND QUALIFICATIONS IRELAND

EXAMINATION

PROBABILITY AND STATISTICS

Lecturer(s):
External Examiner(s):

Date: XXXXXXXX **Time:** XXXXXXX

THIS PAPER CONSISTS OF FIVE QUESTIONS
FOUR QUESTIONS TO BE ATTEMPTED
SECTION A - COMPULSORY
SECTION B - TWO QUESTIONS TO BE ATTEMPTED
ALL QUESTIONS CARRY EQUAL MARKS
THE USE OF NON-PROGRAMMABLE CALCULATORS IS PERMITTED DURING THIS EXAMINATION
LOG TABLES WILL BE SUPPLIED TO STUDENTS
NORMAL DISTRIBUTION TABLE ATTACHED AT BACK OF EXAMINATION PAPER

SECTION A - COMPULSORY

QUESTION 1

Answer (a), (b) and (c)

(a) A computer program produces random bit strings of length 8. Calculate the probability of the following:

(i) A bit string contains no zeros.

(ii) A bit string either starts or ends with a zero.

(6 marks)

(b) On a throw of two fair dice, the result is given by adding the upper faces on each die. What is the probability that the result is:

(i) < 5

(ii) Less than or equal to 10.

(8 marks)

(c) A certain clinical test is 98% effective in detecting a disease when the disease is in fact present. However, the test also shows a false positive in 1% of cases where the patient tested is in fact healthy. If 0.4% of the population actually suffers from the disease, what is the probability that a person who tests positive actually has the disease?

(11 marks)

Total (25 marks)

QUESTION 2

Answer (a), (b) and (c)

(a) Explain, with the help of an example, the difference between a control group and a treatment group. What precisely, is the role of the control group?

(6 marks)

(b) Outline the difficulties associated with convenience sampling. Provide an example to support your answer. Given its difficulties, is there ever any argument to be made for the use of such an approach?

(8 marks)

(c) A survey of 800 Irish households finds that 54% of households have oil-fired central heating. Infer a 95% confidence interval for the percentage of the total population of Irish households that have oil-fired central heating.

(11 marks)

Total (25 marks)

SECTION B - TWO QUESTIONS TO BE ATTEMPTED

QUESTION 3

Answer (a), (b) and (c)

(a) Provide an example of a Binomial random variable.

(6 marks)

(b) Calls at a call centre are observed to arrive at a mean rate of two per minute. Assuming a Poisson distribution, what is the probability of receiving four calls in a minute?

(8 marks)

(c) A study of data collected at a tyre factory shows that a batch of 8000 tyres have a mean wear life of 35000 km with a standard deviation of 7000 km. Assuming a Normal Distribution, estimate:

(i) How many tyres will wear out before 30000 km?

(5 marks)

(ii) How many tyres will continue for more than 45000 km before wearing out?

(6 marks)

Total (25 marks)

QUESTION 4

Answer all parts

(a) A software development team of four must be chosen from three women and four men. In how many ways can two men and two women be chosen?

(6 marks)

(b) On a given afternoon, a delivery company must make three deliveries to separate customers. Five couriers are available. Two of the couriers are trainees.

(i) In how many ways can the couriers be allocated to the deliveries?

(4 marks)

(ii) In how many ways can the couriers be allocated to the deliveries using only experienced couriers?

(4 marks)

(c) A password on a certain computer system is known to be composed entirely of upper or lower case alphabetic letters without repetition.

(i) If the password is eight characters long, what is the probability of randomly guessing the password?

(5 marks)

(ii) Even given the low probability of randomly guessing a password with this approach it is unlikely to be viewed as secure. Explain how this approach is lacking.

(6 marks)

Total (25 marks)

QUESTION 5

Answer both (a) and (b)

(a) An examination contains twelve multiple choice questions. Each question has five options only one of which is selected by the examination candidate. Calculate the probability that an examination candidate who simply guesses the answers will get at least three questions correct.

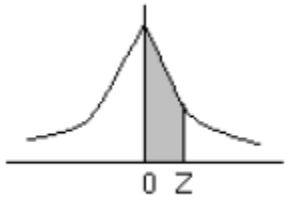
(12 marks)

(b) Single-dose bottles of medicine are stated to have mean contents of 150ml of medicine. There is a suspicion that the bottles are being underfilled and the medicine needs to be taken in a sufficient dosage to be effective. Indeed a random sample of 500 bottles finds that the mean contents are somewhat less, at 148.86ml. With a standard deviation of 8.7ml. Determine whether this result is significant at the 5% level.

(13 marks)

Total (25 marks)

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000